An Integrated Approach for Characterizing and Monitoring the Vadose Zone

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Primary Information (Hard data) : K(☼) and 🗎(☼)

- *Time consuming and labor intensive.*

Secondary Information (soft data): **and**

- Relatively easy to measure by tensiometers, neutron probes, and TDRs in the field.

Take advantage of secondary information to facilitate estimation of unsaturated hydraulic parameters?

INVERSE PROBLEM!

Development of Our Inverse Approach

1. Cokriging (Linear Estimator Approach)

Yeh and Zhang, WRR, 32(9), 1996

Goal:

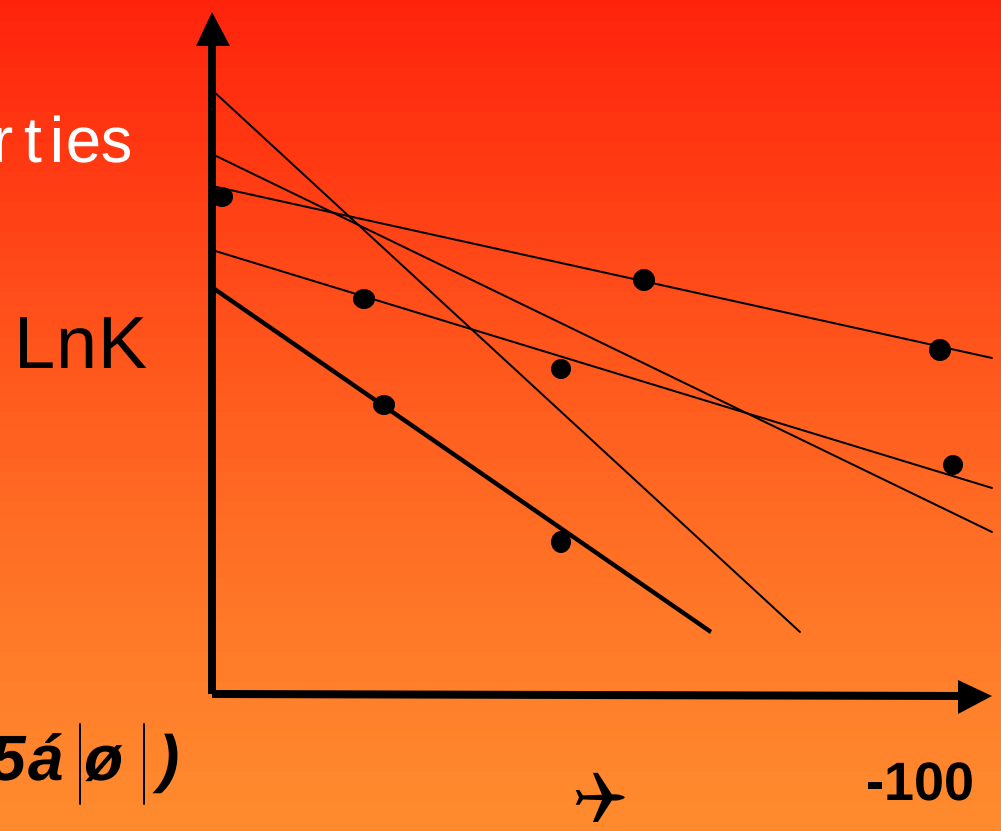
To estimate ***Conditional means*** *of unsaturated hydraulic conductivity parameters based on primary and secondary information*

- Steady infiltration
- Local hydraulic properties

$$K(\phi) = K_s \exp(\alpha\phi)$$

$$\Theta = \frac{(\theta - \theta_r)}{\theta_s - \theta_r}$$

$$= (1 + 0.5 * \alpha|\phi|)\exp(-0.5\alpha|\phi|)$$



- LnK_s and $Ln\alpha$ are stochastic processes in space

Cokriging technique:

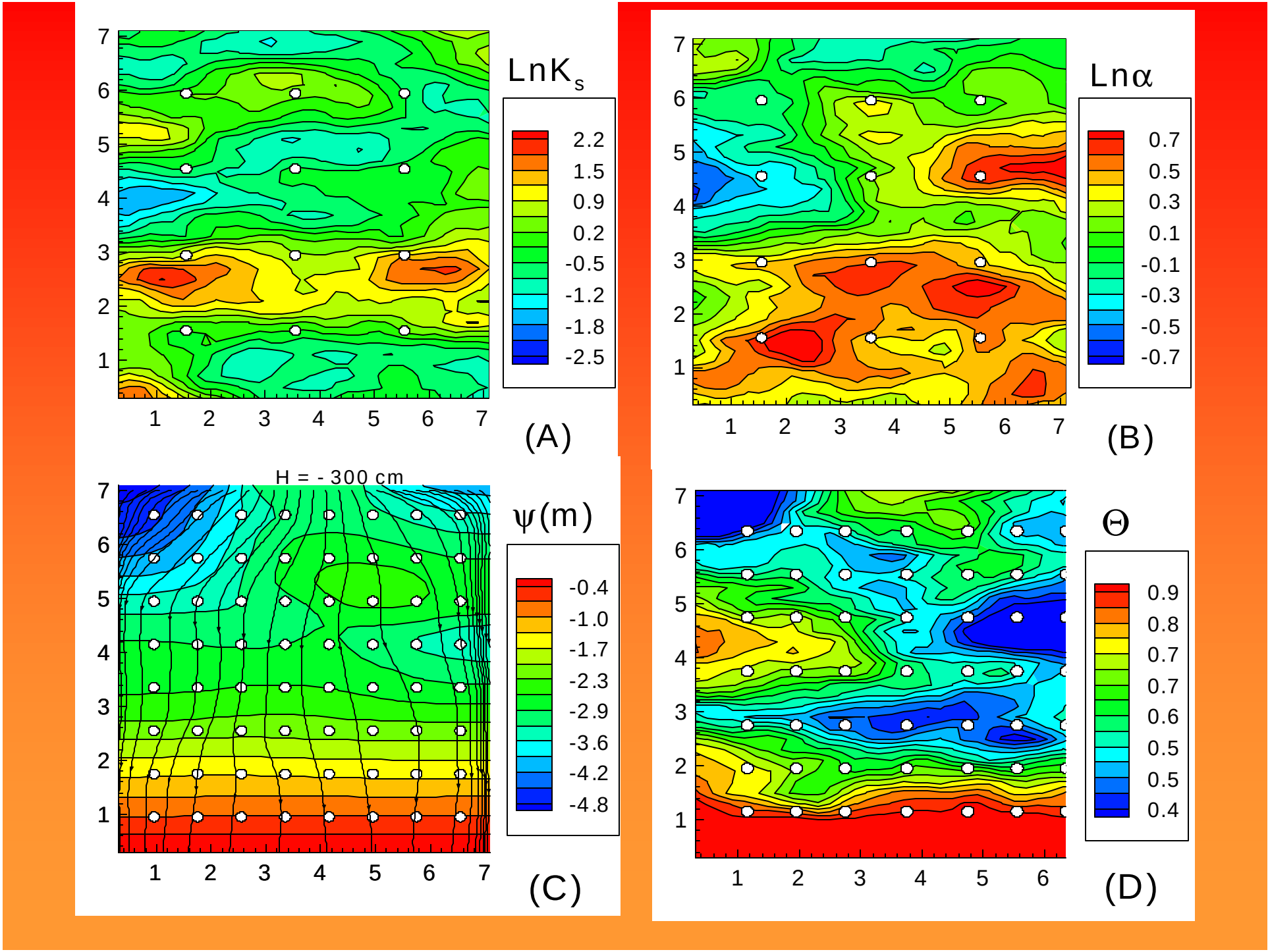
$$Y_c(x_0) = \sum_{i=1}^{n_f} \beta_i \, lnK^*_{s_i} + \sum_{k=1}^{n_h} \lambda_k \, y^*_k + \sum_{l=1}^{n_s} \mu_l \, Q^*_l$$

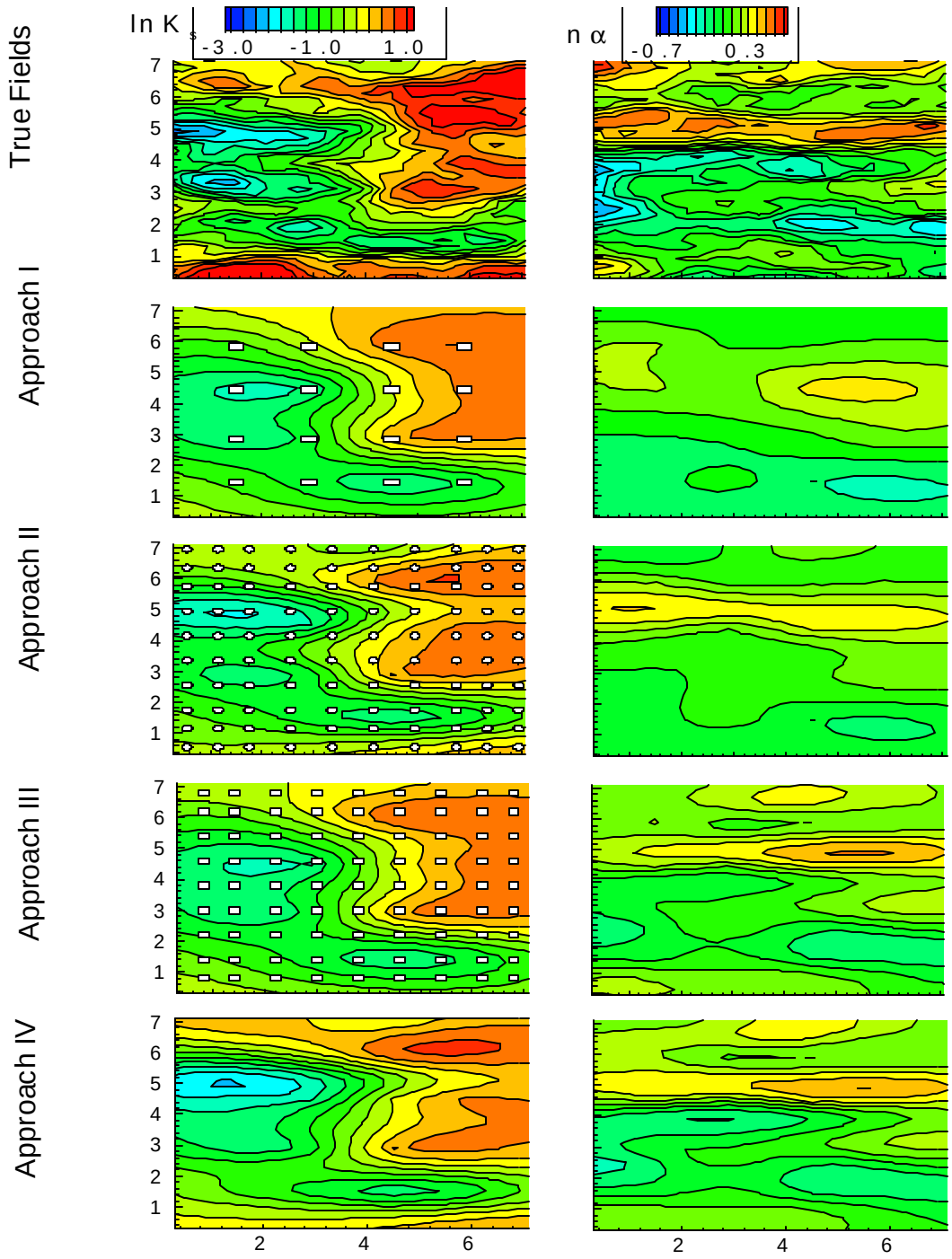
$$Y_c(x_0) = \langle lnK_s(x_0) \rangle_c$$ *Conditional mean*

lnK$_s$, ln✂*, ☼*, and 🗁* = measurements.*

🖇, 🖱, and μ are cokriging weights, depending on

1) correlation functions of lnK$_s$ and ln✂.
2) cross-correlation between lnK$_s$ and ☼, lnK$_s$ and 🗁, ln✂ and ☼, and ln✂ and 🗁.


LnK$_s$
2.2
1.5
0.9
0.2
-0.5
-1.2
-1.8
-2.5
(A)
Lnα
0.7
0.5
0.3
0.1
-0.1
-0.3
-0.5
-0.7
(B)
H = - 300 cm
ψ(m)
-0.4
-1.0
-1.7
-2.3
-2.9
-3.6
-4.2
-4.8
(C)
Θ
0.9
0.8
0.7
0.7
0.6
0.5
0.5
0.4
(D)



True fields

lnKs & ln□

lnKs + ln□ +□

lnKs + ln□ +□

lnKs + ln□ +□+□

What we learn:

- *Secondary information, ☼ and 🗁, improves the estimates of $\ln K_s$ and ln✂.*

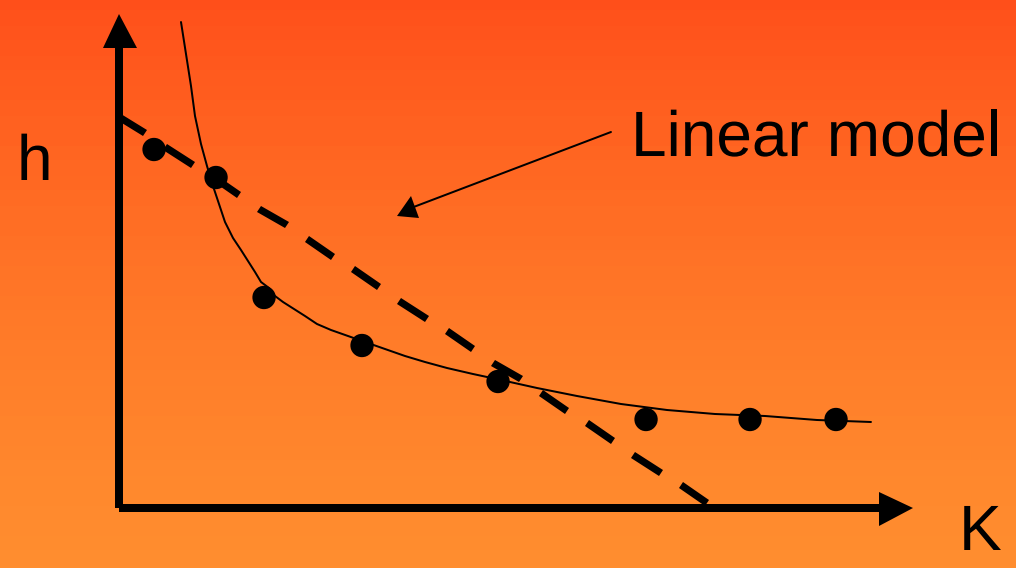
- *Improvements due to ☼ and 🗁 measurements depend on saturation of the soil.*

- *Near saturation, ☼ improves the estimate of $\ln K_s$. Under dry conditions, 🗁 measurements improve the estimate of ln✂.*

- *Cross-correlations between ☼, 🗁, $\ln K_s$, and ln✂ vary with the degree of saturation.*

Limitations:

Linear predictor ***Does not take full advantage of the secondary information.***



Does not honor the head measurements

2. Iterative Geostatistic Inverse Approach

Successive Linear Estimator (SLE)

Zhang and Yeh, WRR 33(1),63-71, 1997

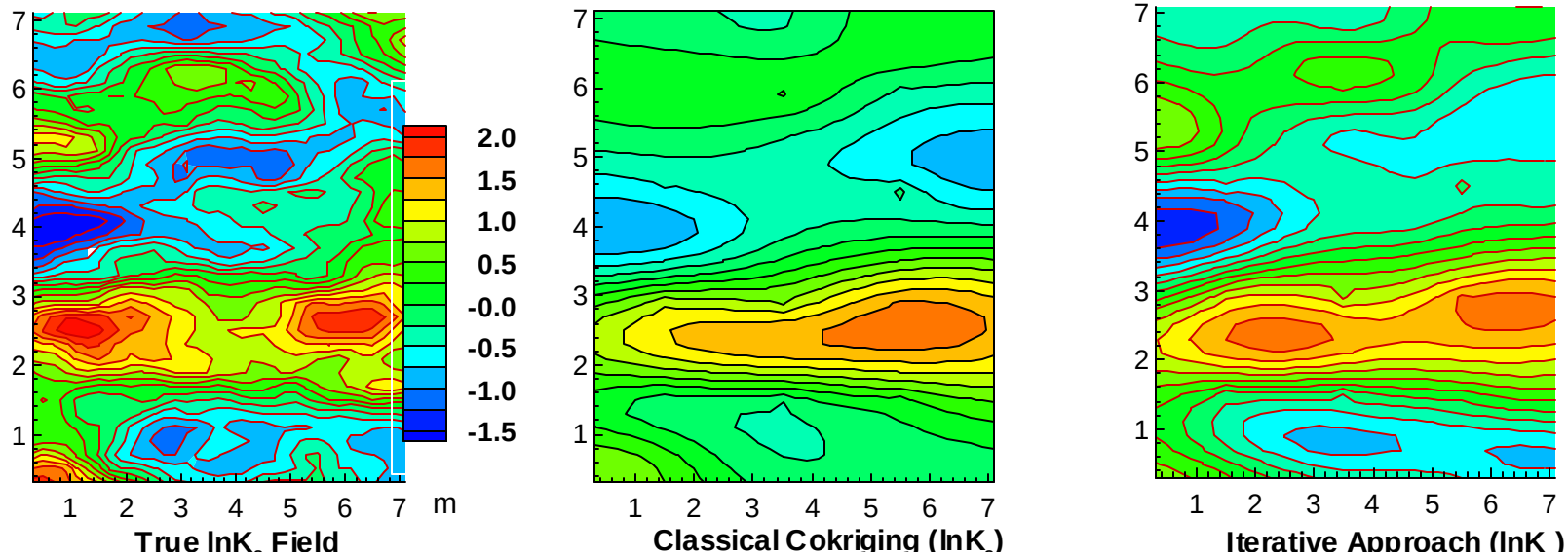
$$Y_c^{(r+1)}(x_0) = Y_c^r(x_0) + \sum_{k=1}^{n_h} l_k^r \left[y_k^*(x_k) - y_k^{(r)}(x_k) \right] + \sum_{l=1}^{n_s} m_l^r \left[q_l^*(x_l) - q_l^{(r)}(x_l) \right]$$

☼ * = *the observed head at sample locations.*

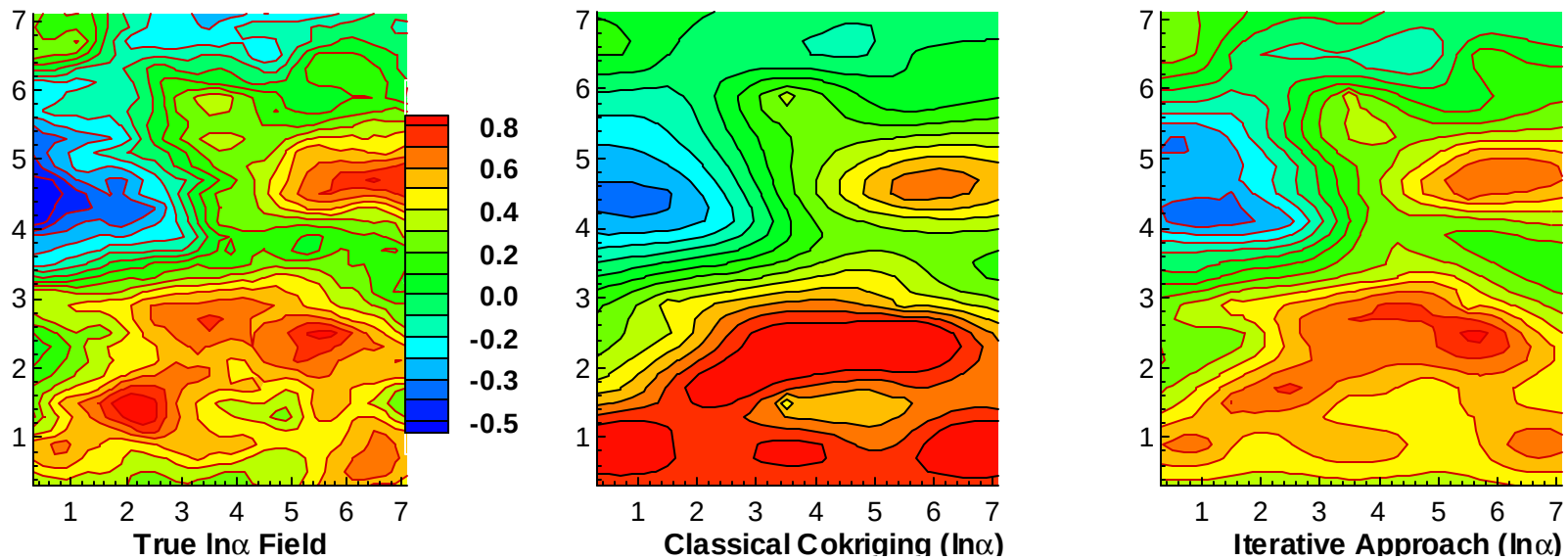
🗁 *= *the observed effective saturation.*

☼ ^{r} *and* 🗁 ^{r} *= simulated head and saturation at the r* th *iteration.*

🖱$_k$, *and* 💻$_l$, *are weighting coefficients at each iteration.*



Contour maps of true and estimated lnK$_s$ and lnα fields
(H=-3.0 m, σ_f^2=1.0, σ_a^2=0.1, and Γ=1.0/m)

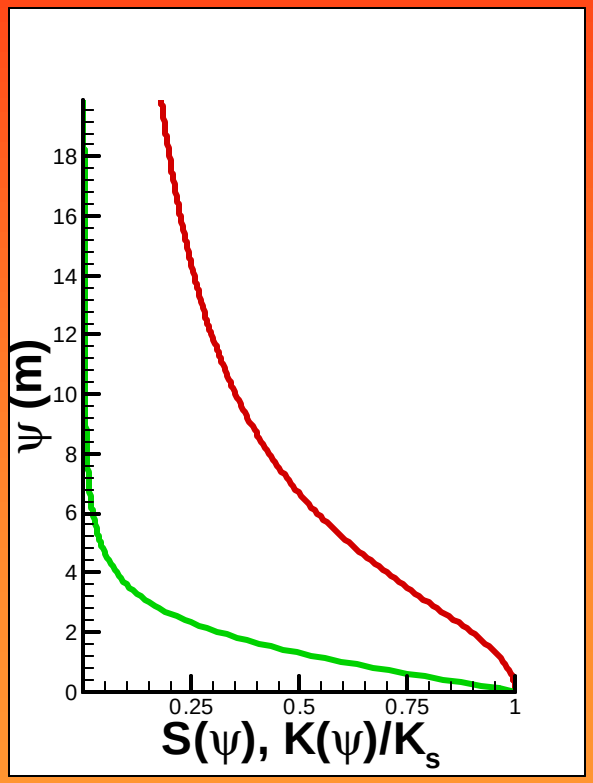


Three-Dimensional, Transient Flow (Hughson and Yeh, WRR, 36(2?), 2000)

- Richards equation

$$\left(S_s b + C(y)\right)\frac{\partial y}{\partial t} = \nabla \cdot \left(K(y)\nabla(y + x_3)\right)$$

- Mualem-van Genuchten model



Saturation

$$S(y) = \left(1 + |a\ y|^n\right)^{\frac{1}{n}-1}$$

Relative hydraulic conductivity

$$K(y) = K_s\sqrt{S(y)}\left[1 - \left(1 - S(y)^{\frac{n}{n-1}}\right)^{1-\frac{1}{n}}\right]^2$$

Moisture Capacity

$$C(y) = a\ (n-1)(q_s - q_r)S^{\frac{n}{n-1}}\left(1 - S^{\frac{n}{n-1}}\right)^{1-\frac{1}{n}}$$

Map view of sampling well locations

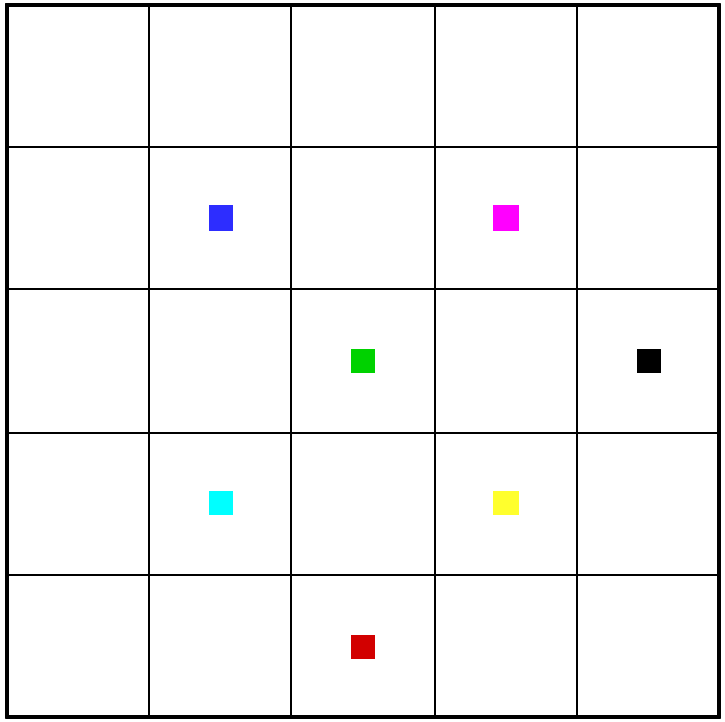


Figure 5a

FEM grid and the 42 sample locations

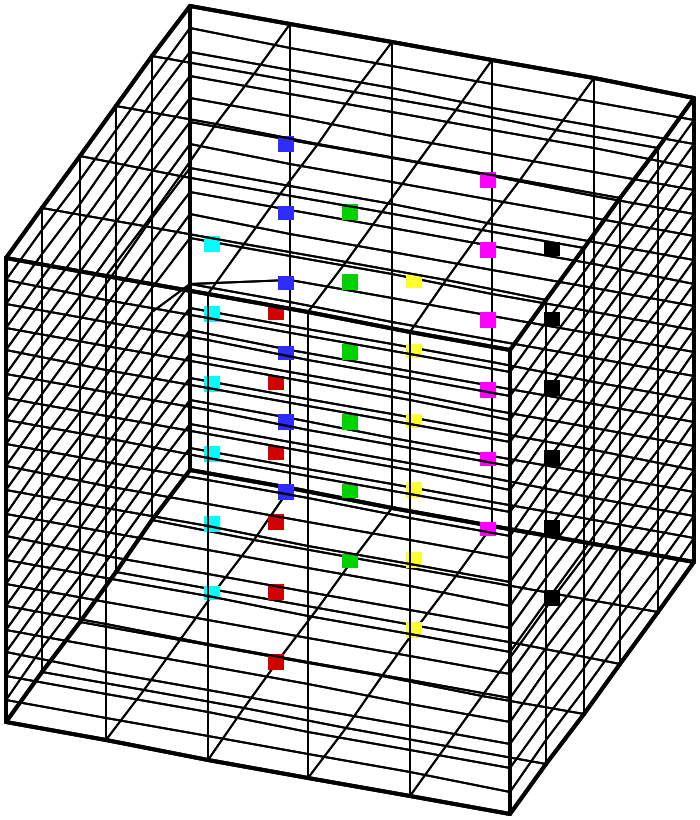


Figure 5b

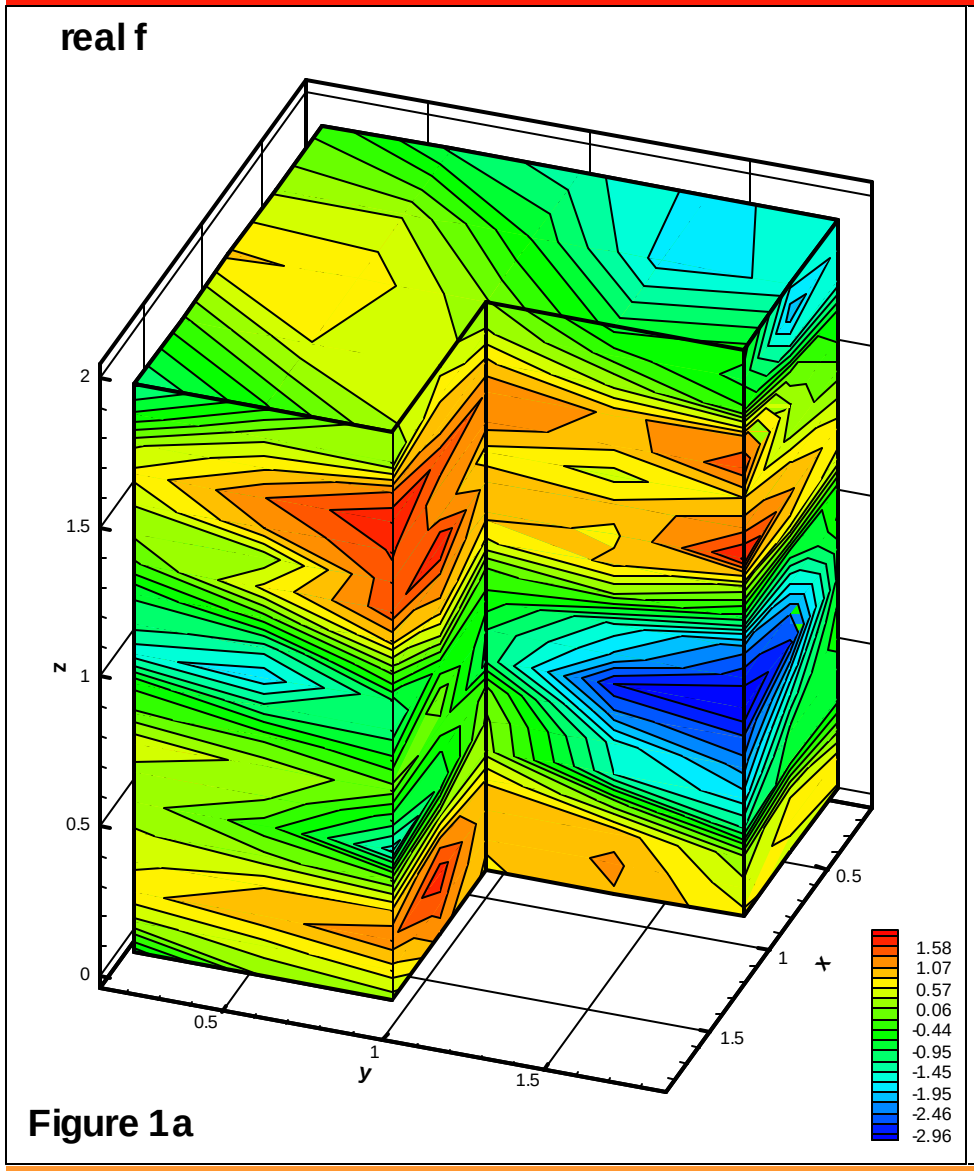


Figure 1a

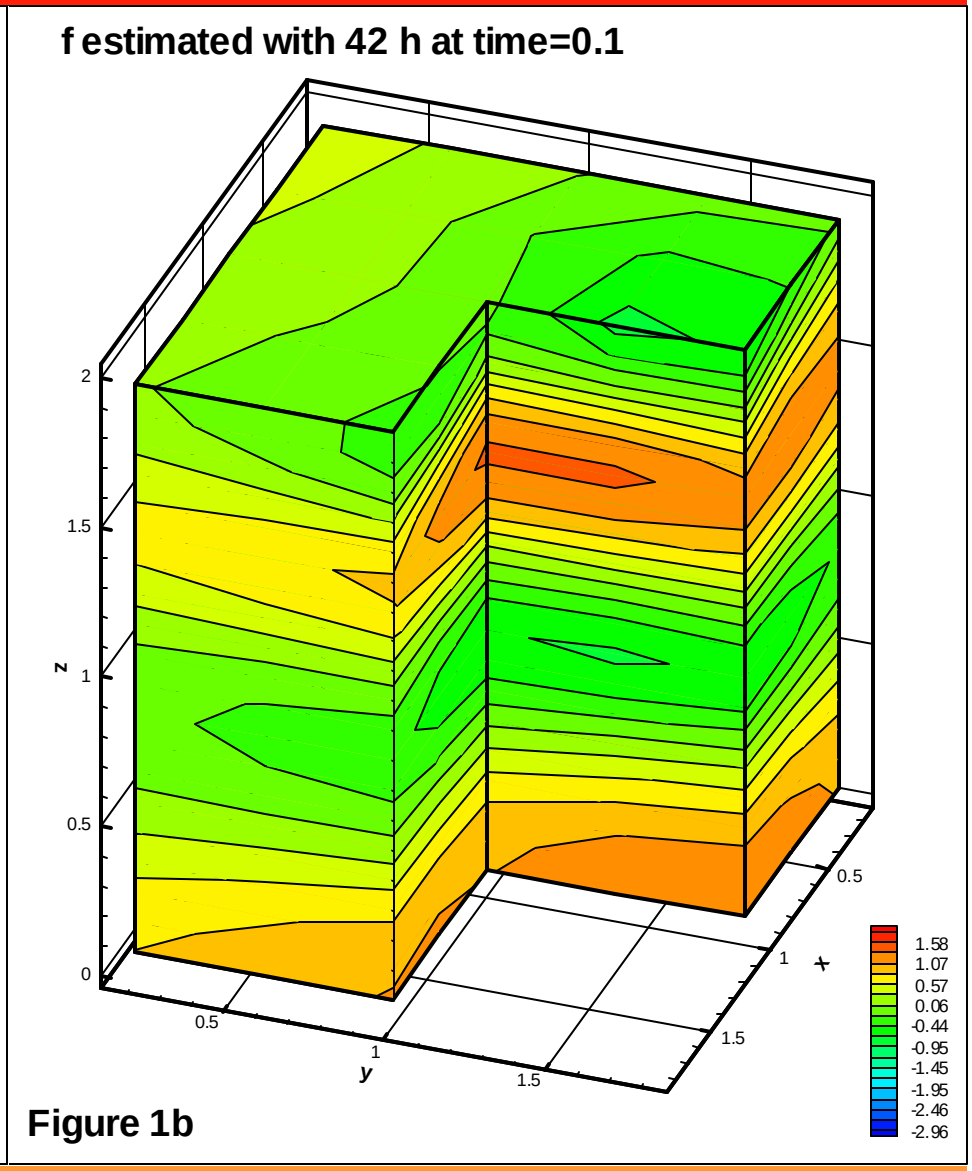


Figure 1b

f estimated with 42 h at time=0.5

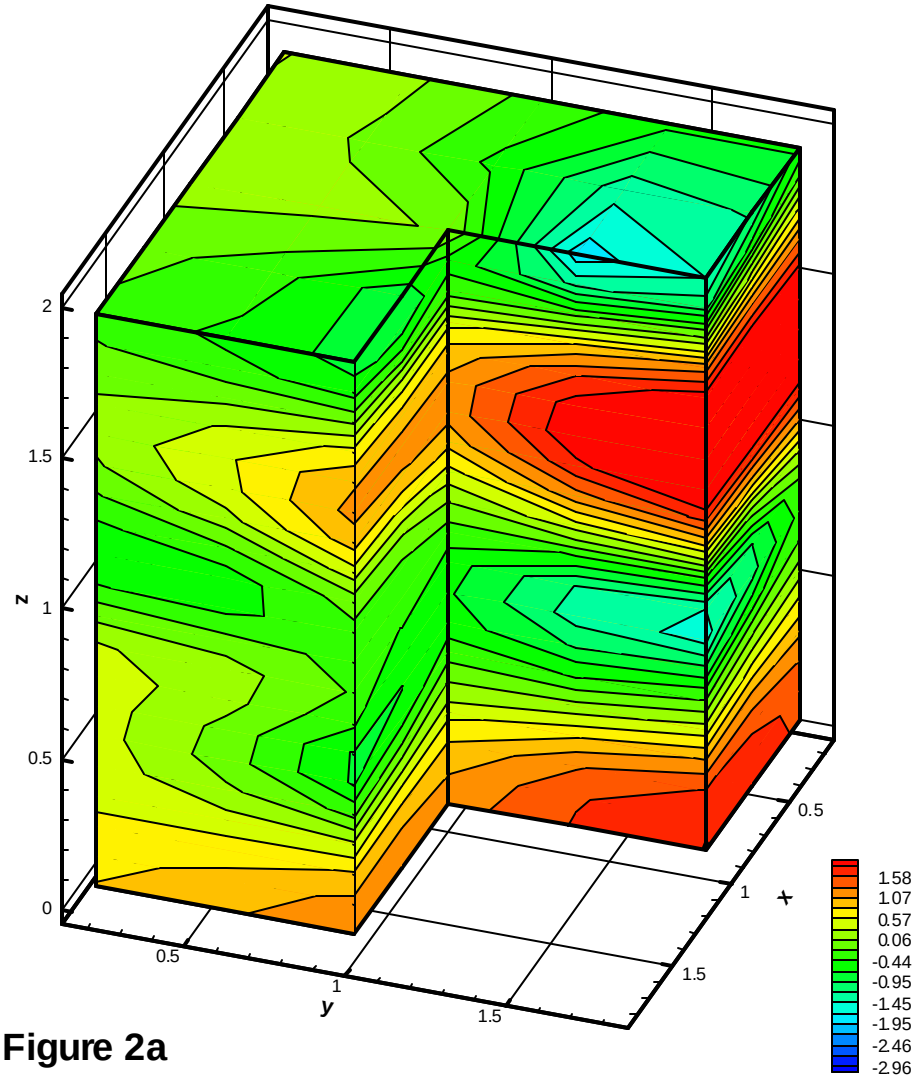


Figure 2a

f estimated with 42 h at time=1.0

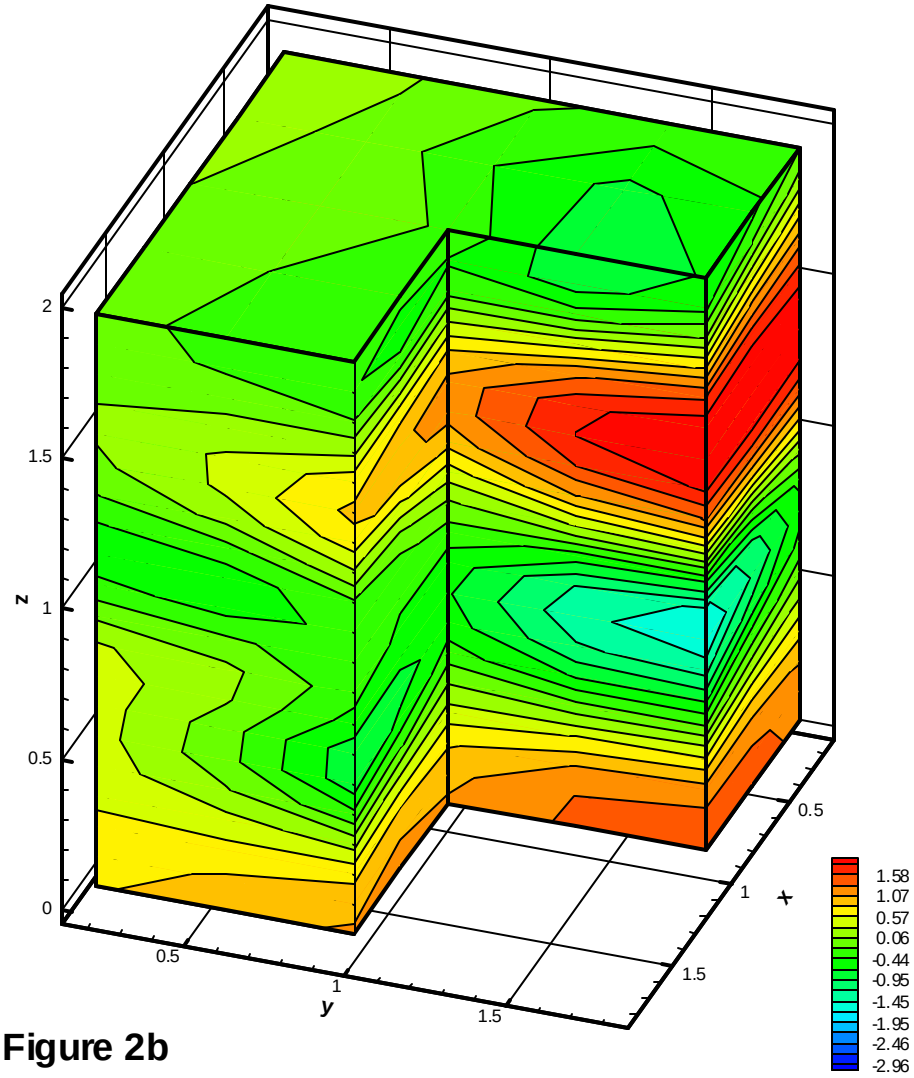


Figure 2b

f estimated with 42 h sequentially at times=.1, .5. 1.0

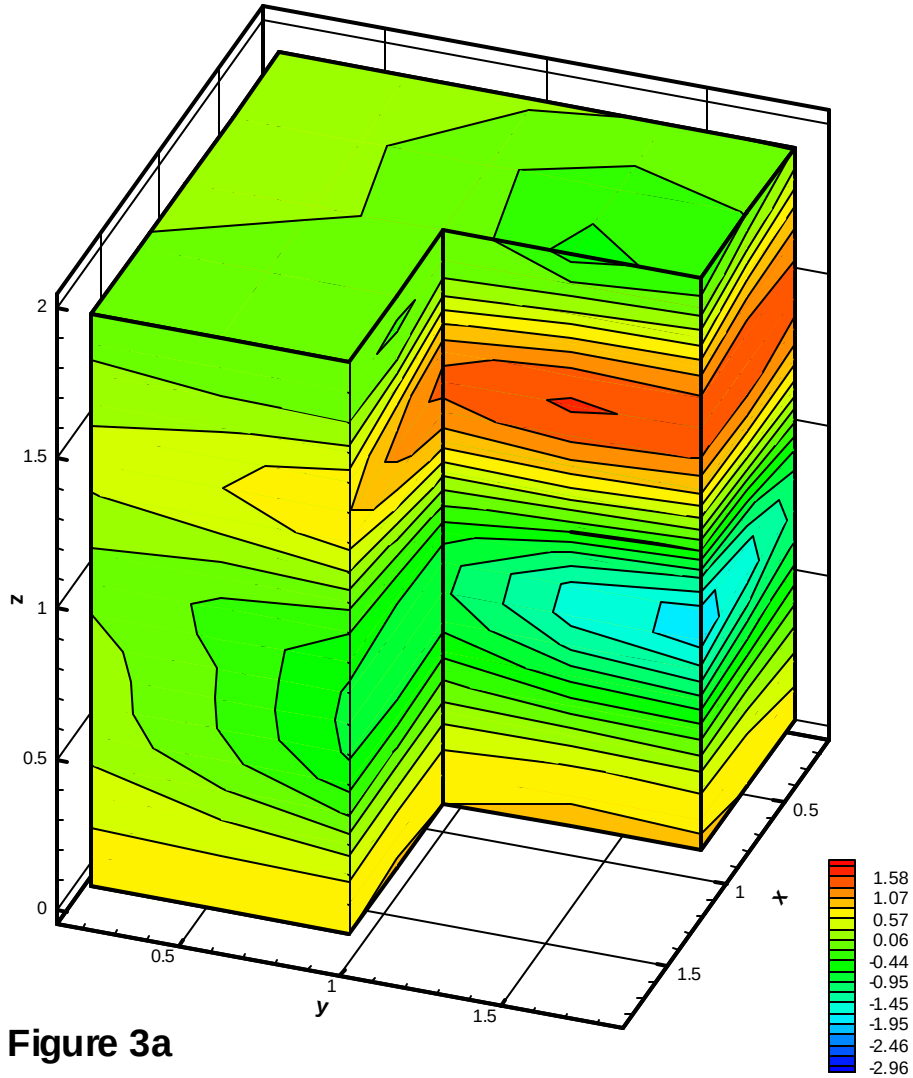


Figure 3a

f estimated with 42 h at steady state

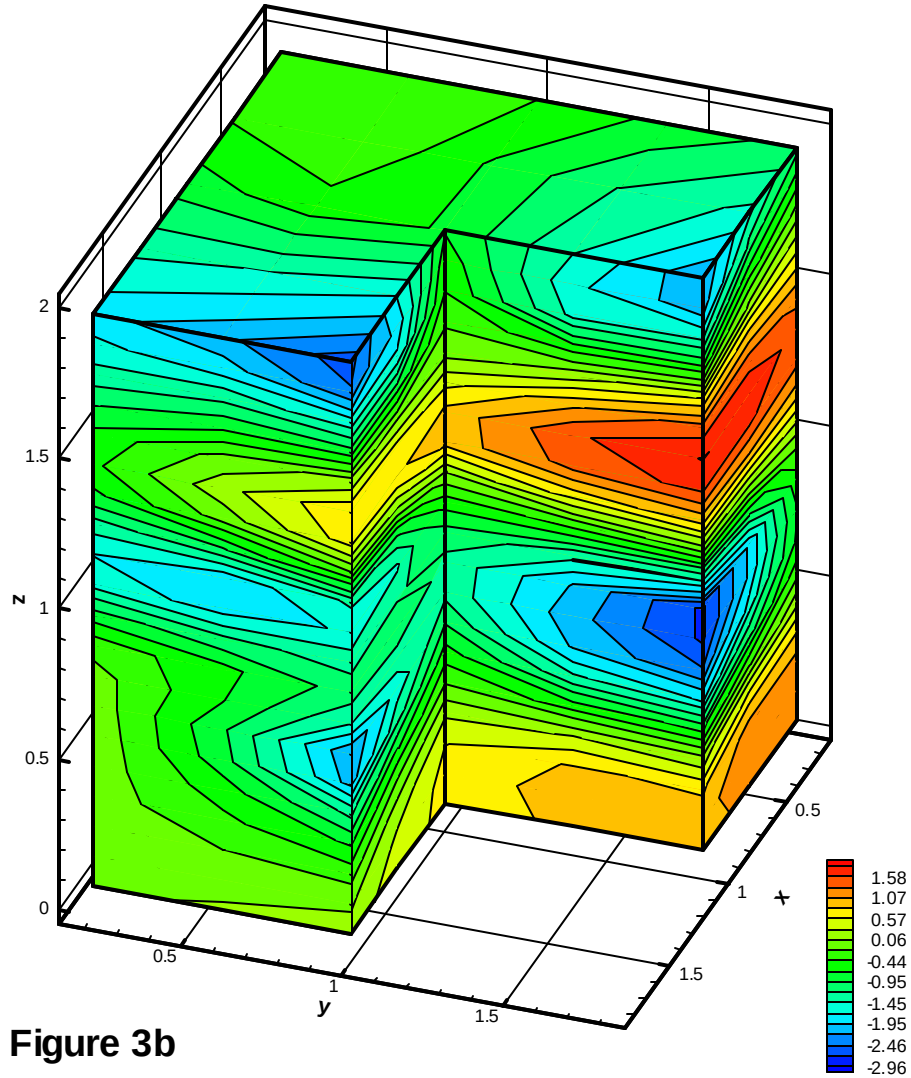


Figure 3b

How does the iterative geostatistical approach work?

Common Sense Approach:

Coarse (general) to fine (specific) description:

1. Covariance functions define the generic pattern of heterogeneity in a given field site.

2. A few measurements of primary information tailor the generic pattern to a site-specific one.

Hard data conditioning!

3. The site-specific one is then improved by the secondary information due to its cross-correlation with the primary variables.

Soft data conditioning!

4. Successive Linear estimator circumvents problems of the linear predictor and reveals more detailed heterogeneity.

Limitations:

A large number of measurements of secondary information (pressure and moisture content) are needed.

How to collect them without drilling a large number of holes?

Geophysics!

Electrical Resistivity Tomography (ERT)

Senses "changes" in "moisture content" and "concentration" over a large volume of geological media.

Really?

Problems associated with ERT

1. Current ERT relies on the classical inversion

No unique solution!
Do not honor point measurements!
What do you estimate?
Numerical artifacts?

2. Spatial variability of resistivity-moisture content relationship

Uncertainty in relating the change in resistivity to change in moisture content.

Socorro field experiment

Alumbaugh, Brainard, Glass, LaBrecque

Layout

Animation

Tensiometer data

Neutron probe data

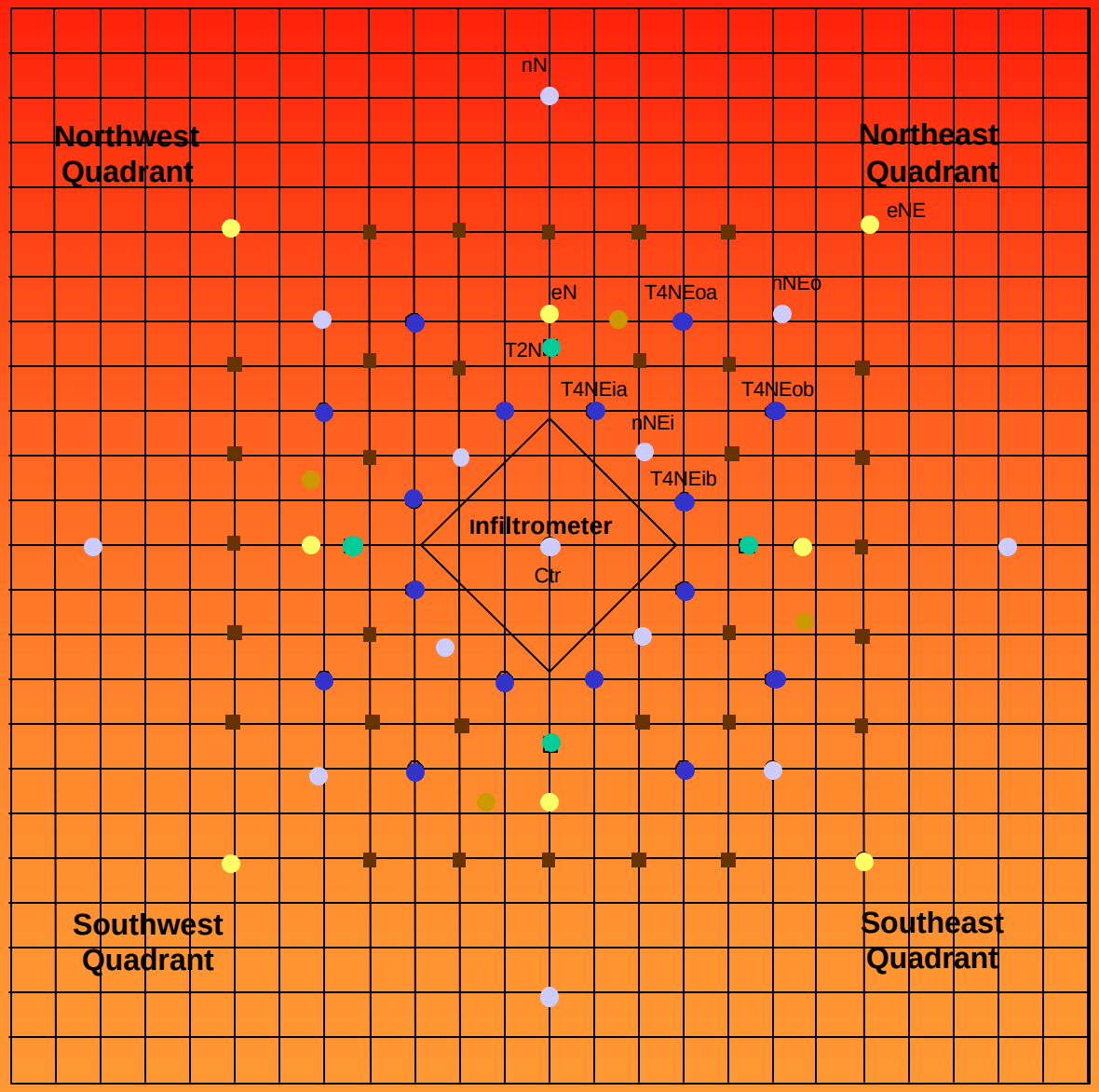
ERT /xbGPR data

Difficulties

HHGIT Instrument Layout - Plan View

The nomenclature for identifying instrument and access tube locations is given for the NE array. Location identifiers for the other arrays are obtained by rotating the NE quadrant array 90 degrees about the center access tube and changing the quadrant abbreviation to specify the new quadrant.

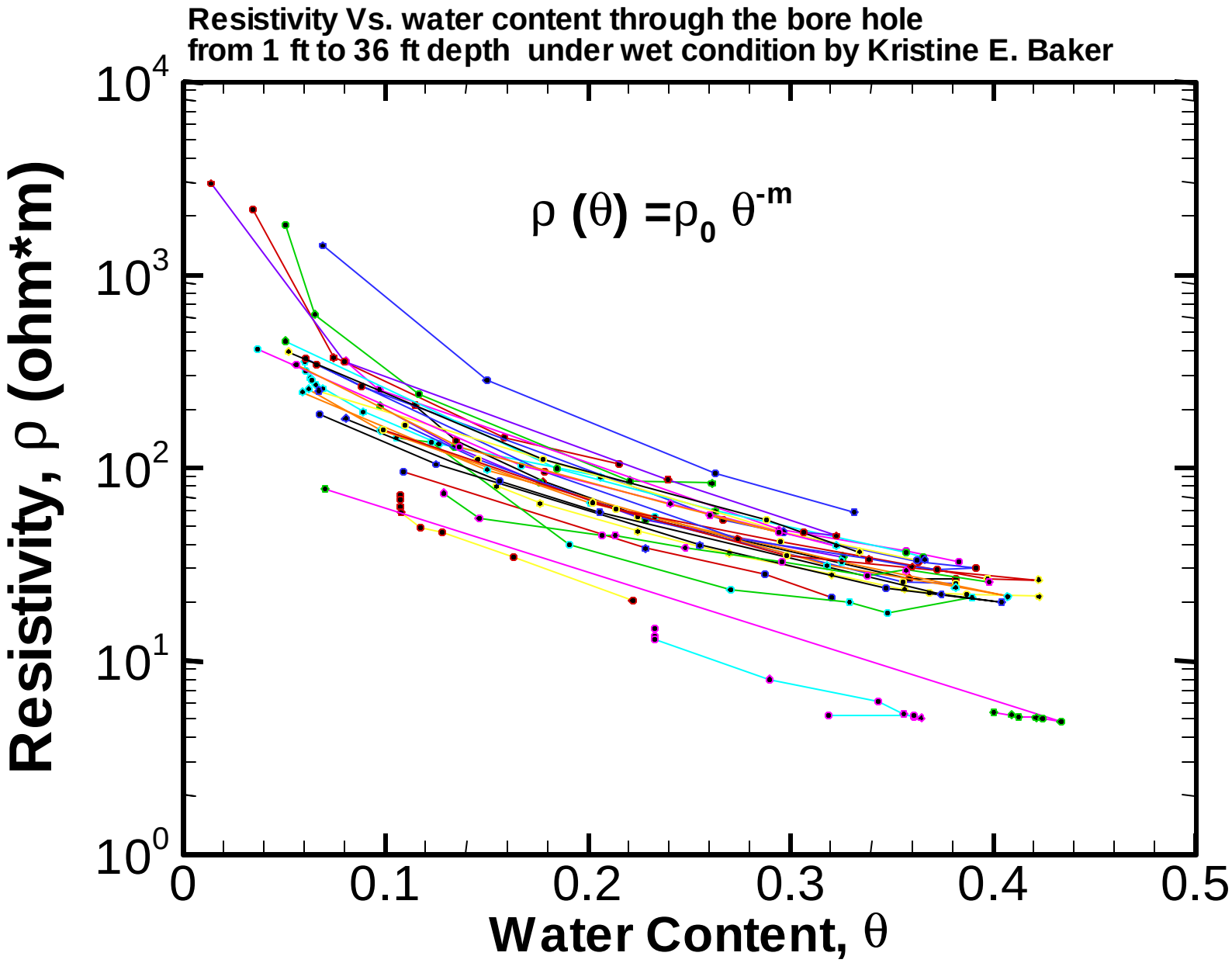
- Four-Nested Tensiometer Locations (T4 ID prefix). The tensiometer porous cups are approximately 2, 4, 6, and 8 meters below the ground surface/datum. A TDR probe is installed at the bottom of each hole with a suction lysimeter immediately above. The TDR probe/suction lysimeter pair is located just below deepest tensiometer porous cup.
- Two-Nested Tensiometer Locations (T2 prefix). Each hole has two instrument clusters consisting of a tensiometer, a suction lysimeter, and a TDR probe (listed from top to bottom). The clusters are approximately 3.5 and 6 meters below the ground surface/datum.
- PVC Cased Wells (n prefix). These wells are predominantly used for collecting neutron and GPR data.
- ERT electrode string (e Prefix).
- ERT surface electrode.
- Continuous Core Sample Locations.



3m

Scale

N


Resistivity Vs. water content through the bore hole
from 1 ft to 36 ft depth under wet condition by Kristine E. Baker
ρ (θ) =ρ0 θ-m
Resistivity, ρ (ohm*m)
Water Content, θ
10^4
10^3
10^2
10^1
10^0
0
0.1
0.2
0.3
0.4
0.5

Solution?

- *Integration of different pieces of information (circumstantial evidences):*

  Hydrology, soil, geology, geochemistry, geophysics, point measurements….etc.

Conditional Mean

Example, hydrogeophysical joint inversion

Iterative HydroGeophysical Joint Inversion

1. New ERT inversion using our conditioning approach

Integrate with point measurements of moisture content, concentration, and resistivity.

Include geological, moisture, concentration spatial structures

Geologically and hydrologically correct ERT interpretation, Better estimates of moisture content.

2. Hydrological Inversion

Use results from ERT to yield better estimates of unsaturated hydraulic parameters

Better predictions of moisture and concentration distributions that can help ERT

3. Iteration between ERT and Hydrological Inversions

Conditional mean estimates
Uncertainty

A cost-effective site characterization and monitoring tool

Advantages of our Joint Inversion

1. *Uses the reciprocal nature of the hydrological and geophysical information and inversion.*

   maximize the utility of available information

2. *Provides unique solution to hydrological and geophysical inversion:*

   conditional mean estimates

3. *Address uncertainties associated with our estimates*

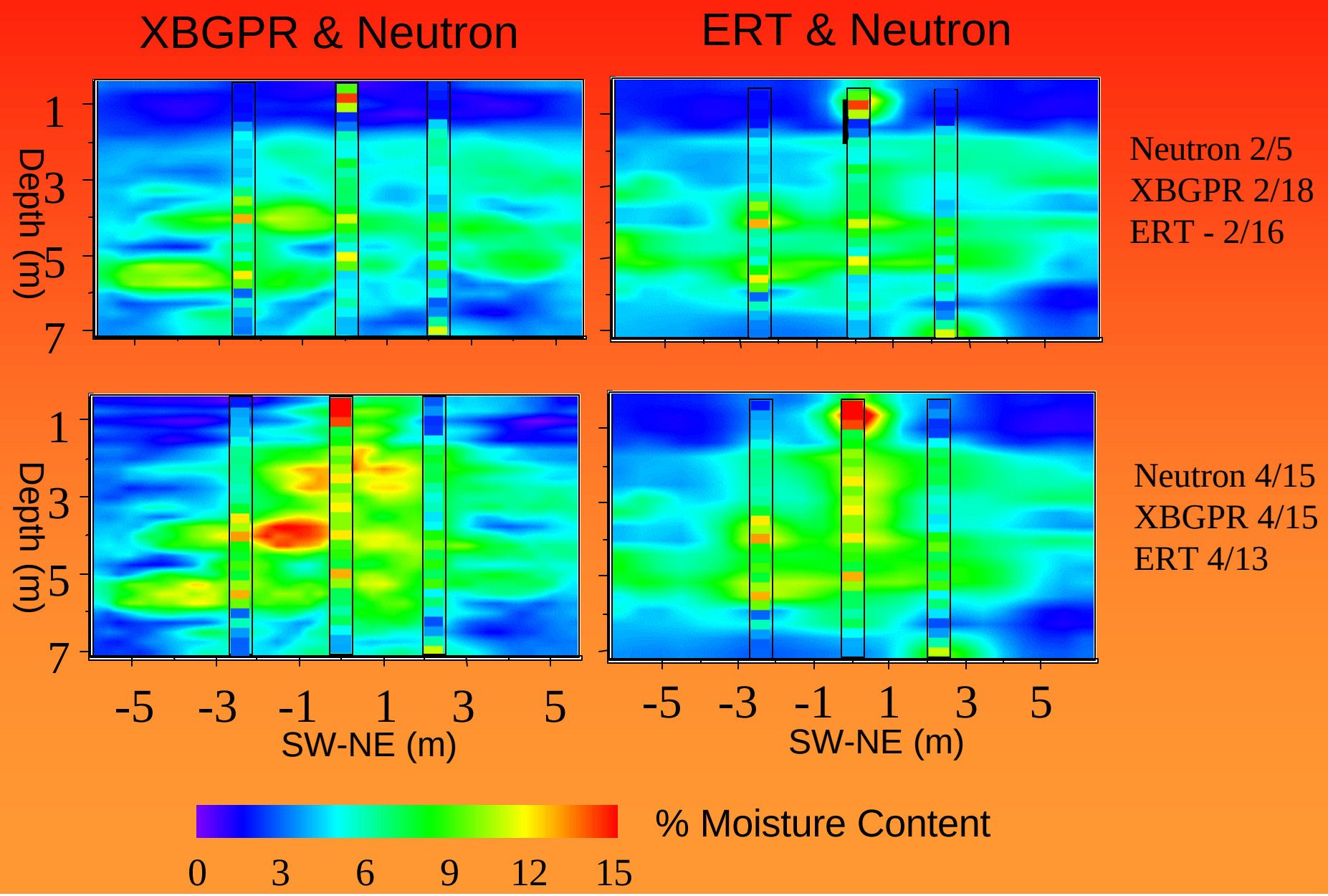
   assessment of remediation operations

The end

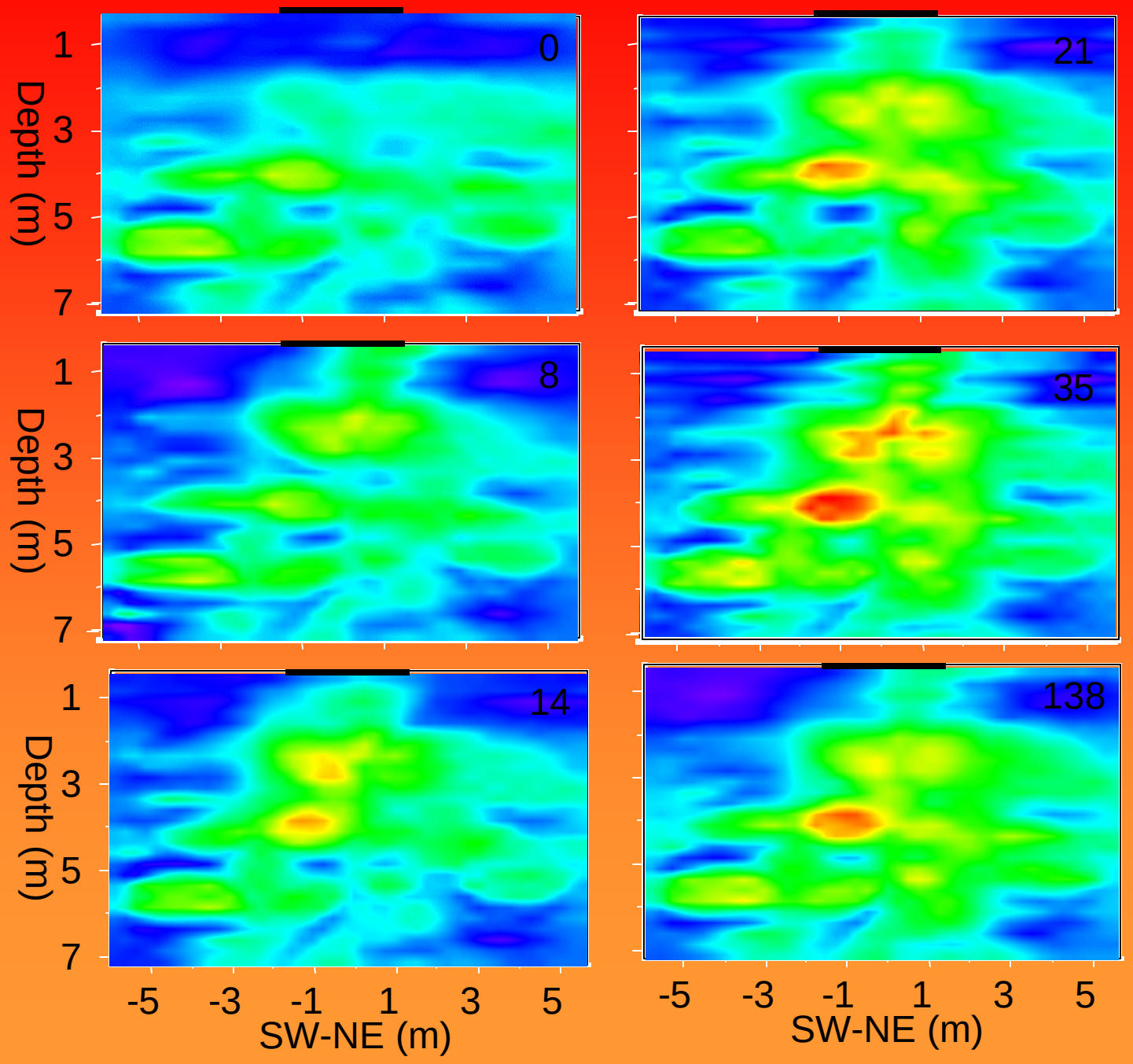
Validation of Imaging Results

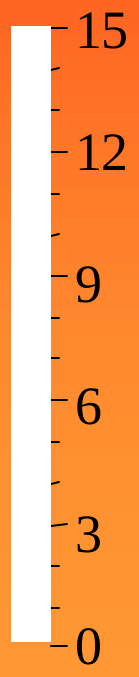
- Can compare ERT and XBGPR to each other.*
- Can compare to geophysical logs collected in PVC cased wells.
  - Neutron Moisture*
  - Electromagnetic Conductivity (only for pre-infiltration)
- Can compare to hydrologic sensors.

ERT, XBGPR and Neutron Moisture



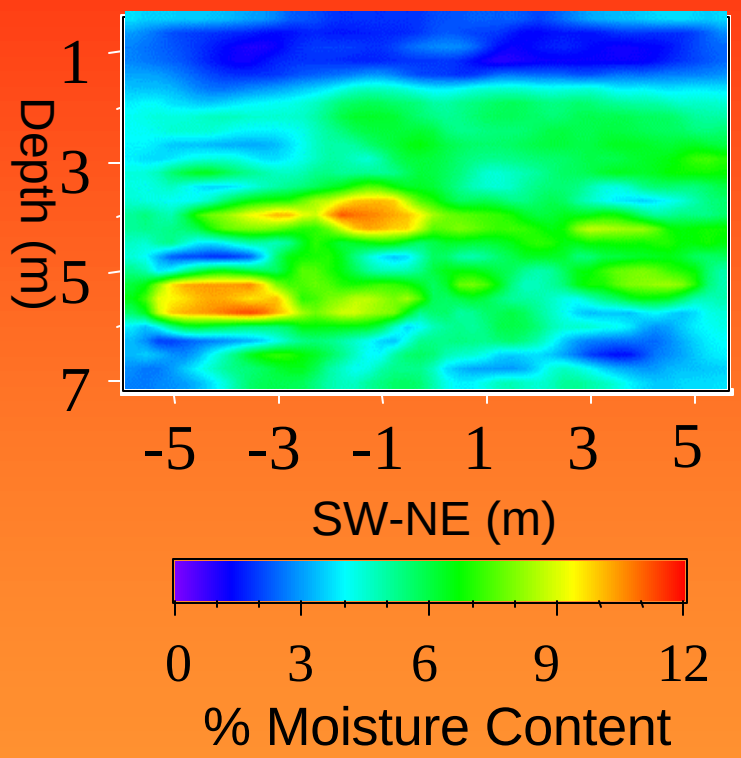
XBGPR 'Raw' Images

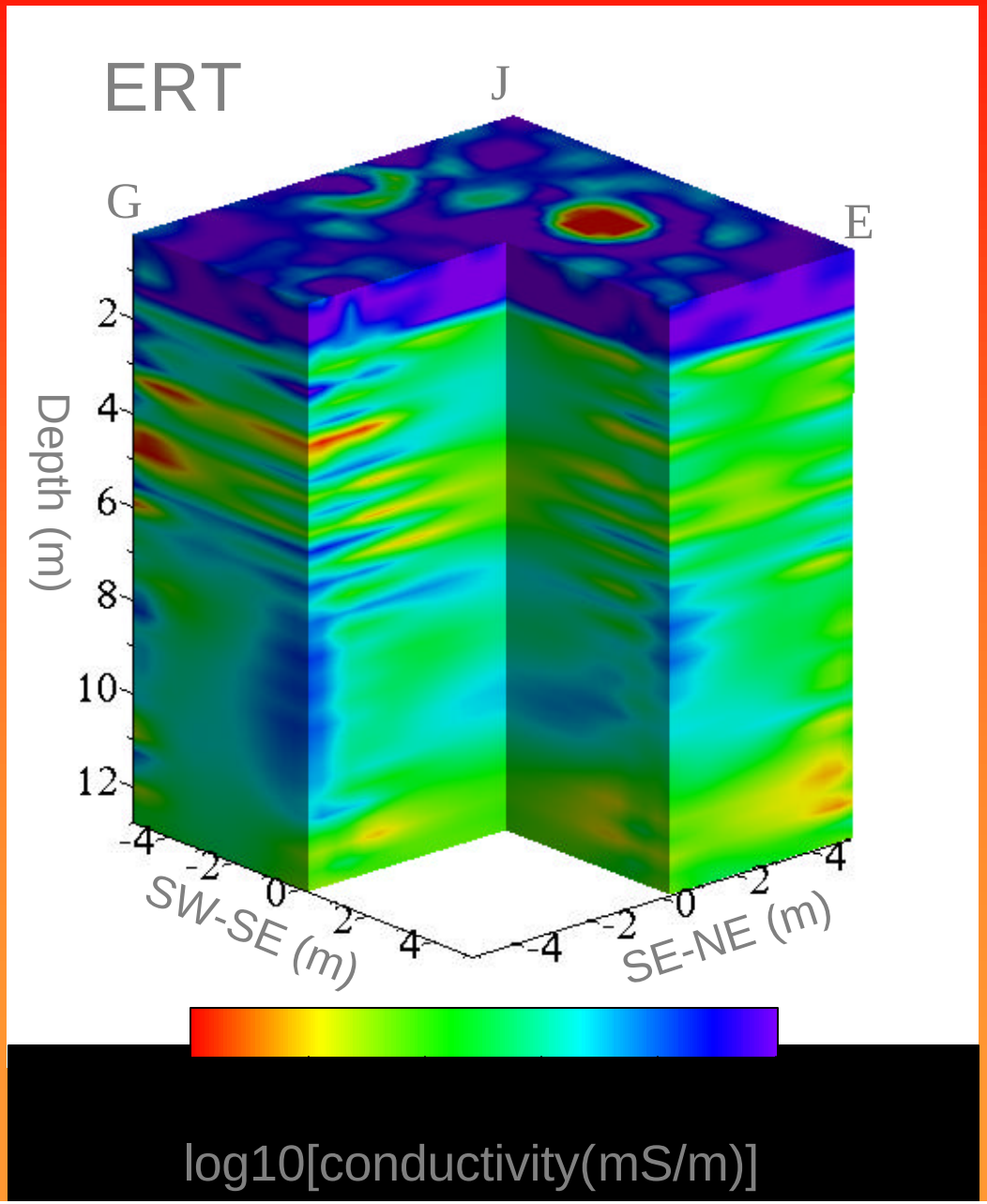




Preinfiltration Images

XBGPR





Infiltration Experiment Specifics

- Started on March 11, 1999
- Infiltrating 2.5cm/day over 3m by 3m area.
  - Flux rate much less than saturate hydraulic conductivity.
  - Infiltrometer active for two 5 minute periods/day.
- ERT data collected daily at first, to once a week now.
- GPR data collected once a week at first, monthly now.

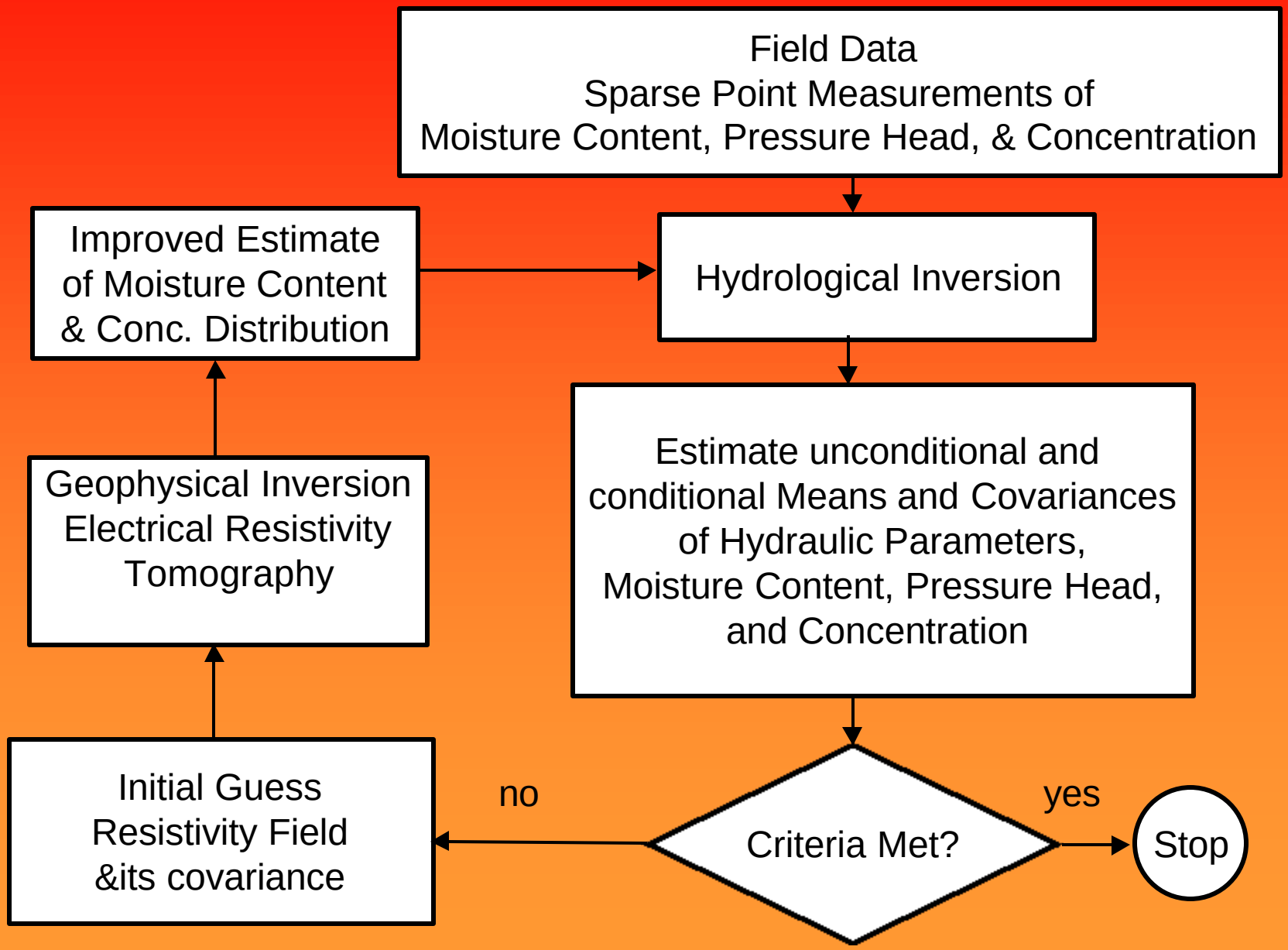
Classical Inverse Approach

Minimum-Output-Error Approaches
(nonlinear regression)

Problems:

- *Non-unique solutions*
- *Measurements are not honored*
- *Identity of the estimate.*
- *Computational burden*
- *Limited resolutions*

Iterative Hydrological/Geophysical Inversion Technique (IHGIT)



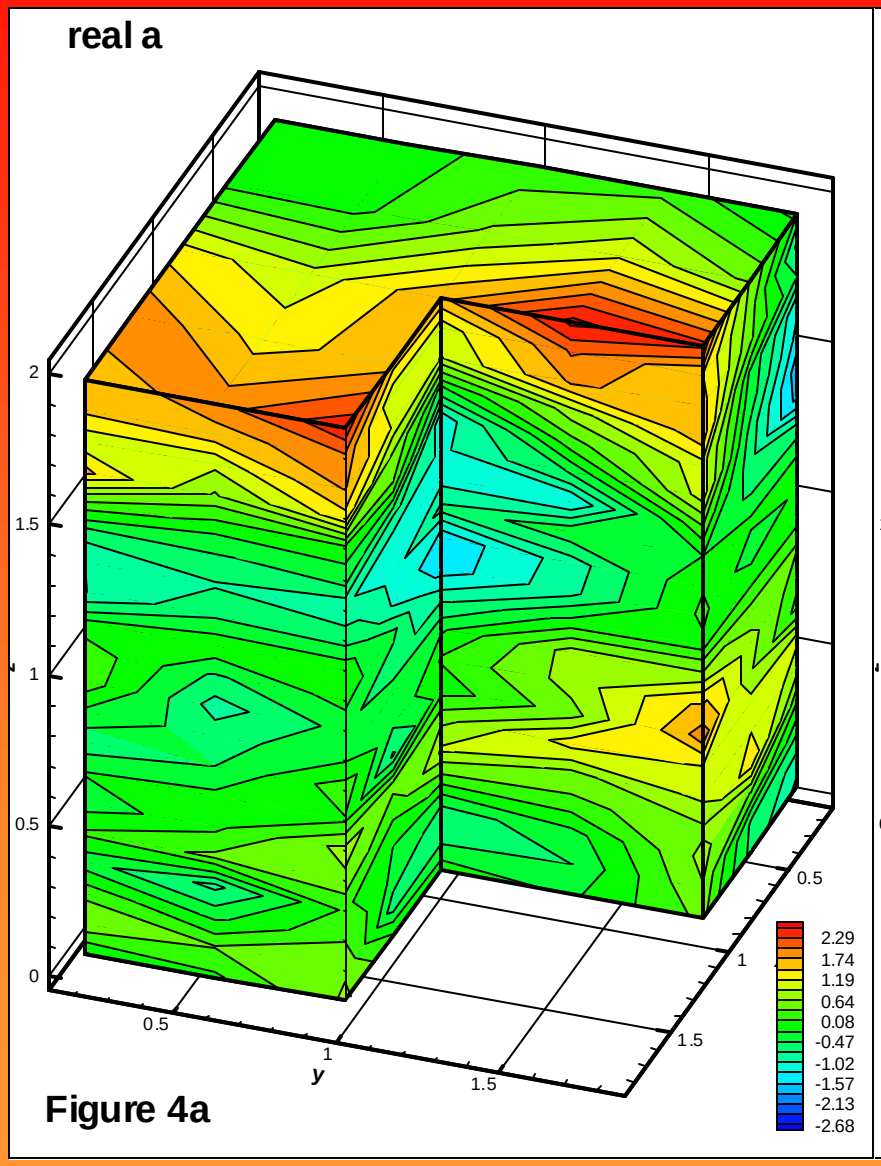


Figure 4a

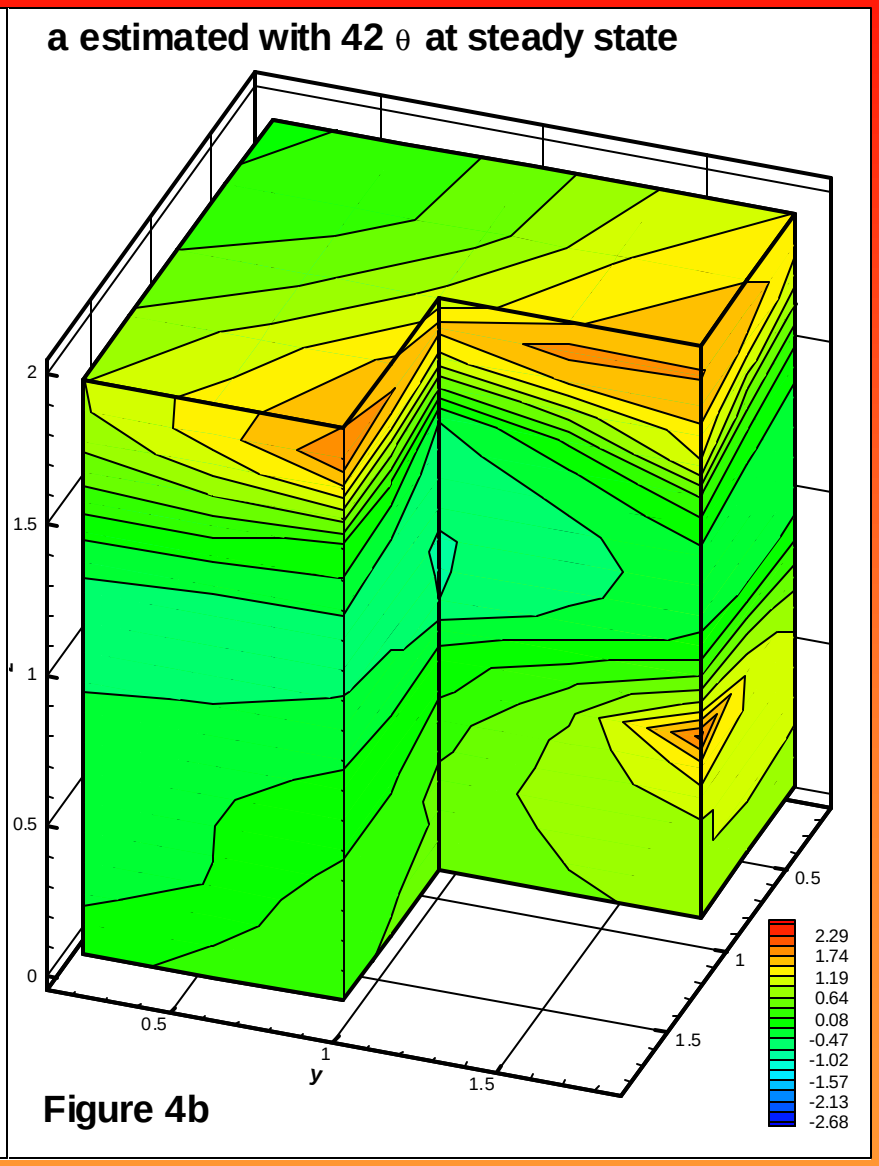


Figure 4b

HIGH RESOLUTION PREDICTIONS

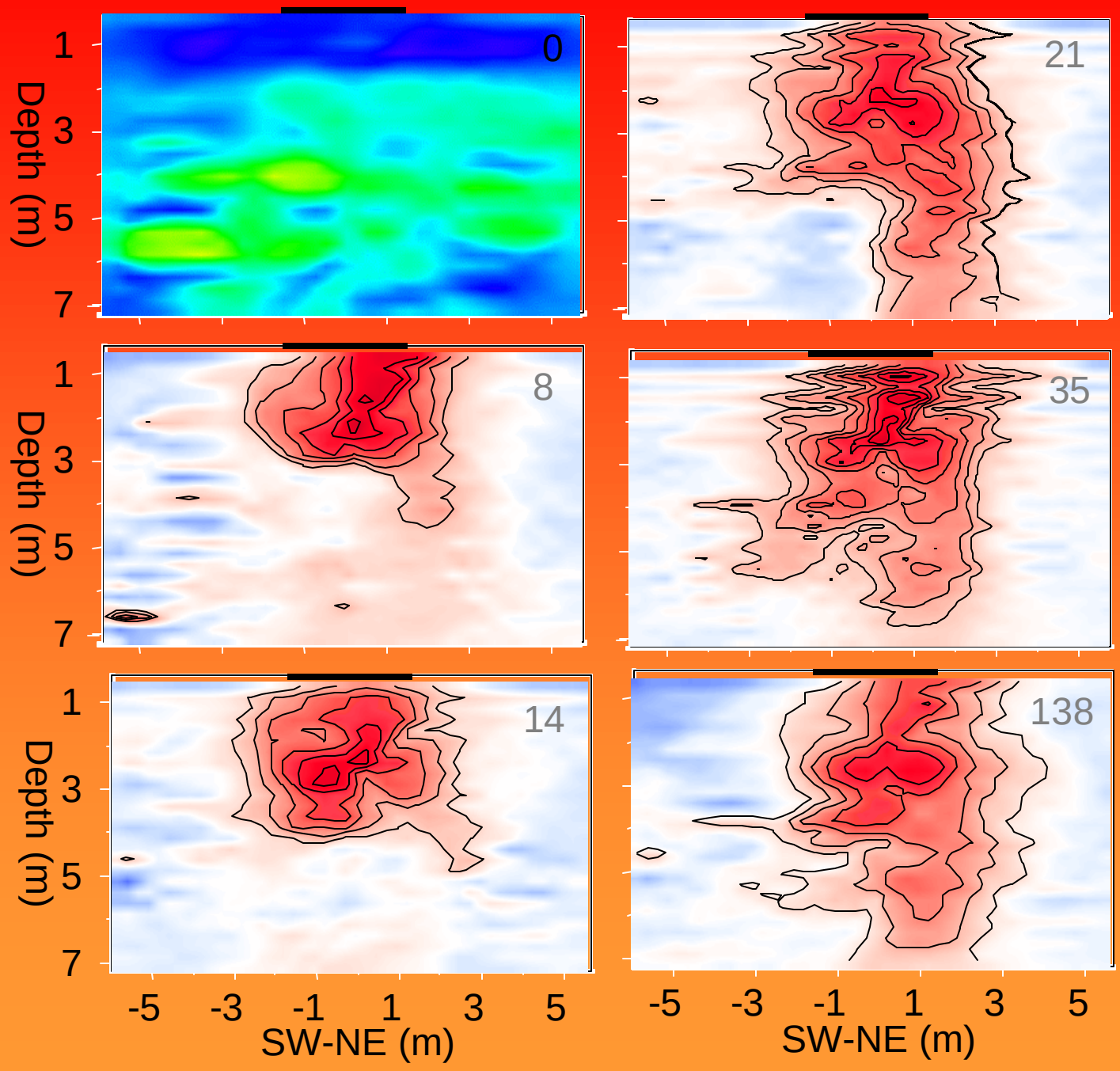
No magic, no panacea

- Detailed description of heterogeneity

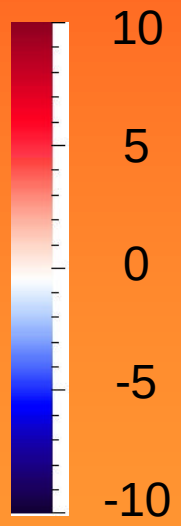
Conditional Mean: (The best we can do)

- Preserve data at sample locations and derive effective or mean values of the parameter at unsampled locations.

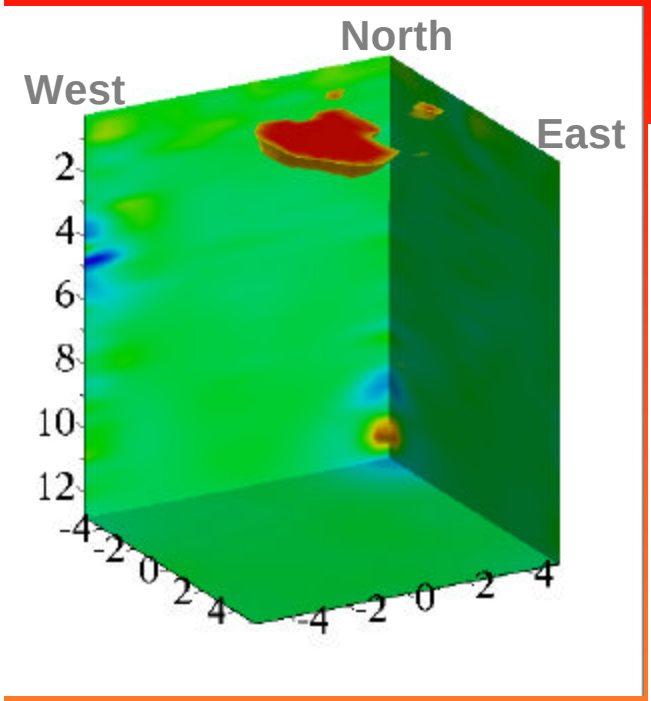
XBGPR Difference Images



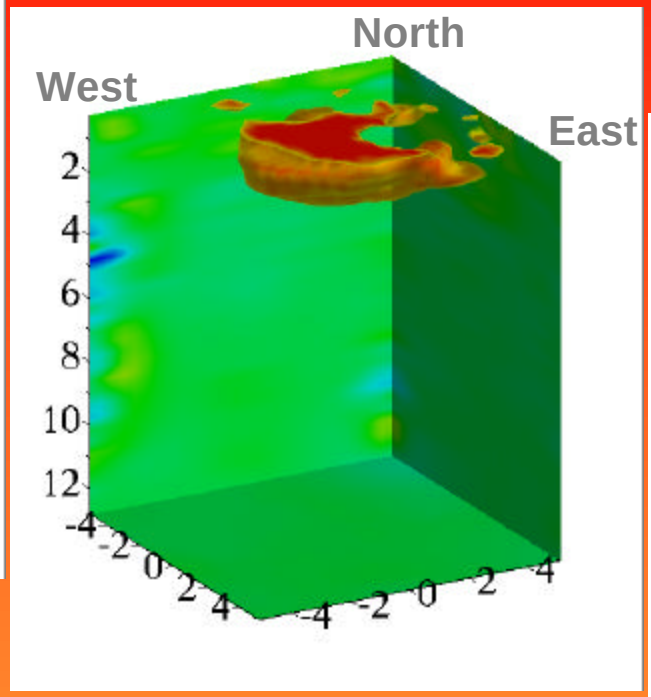
Difference in % Moisture Content



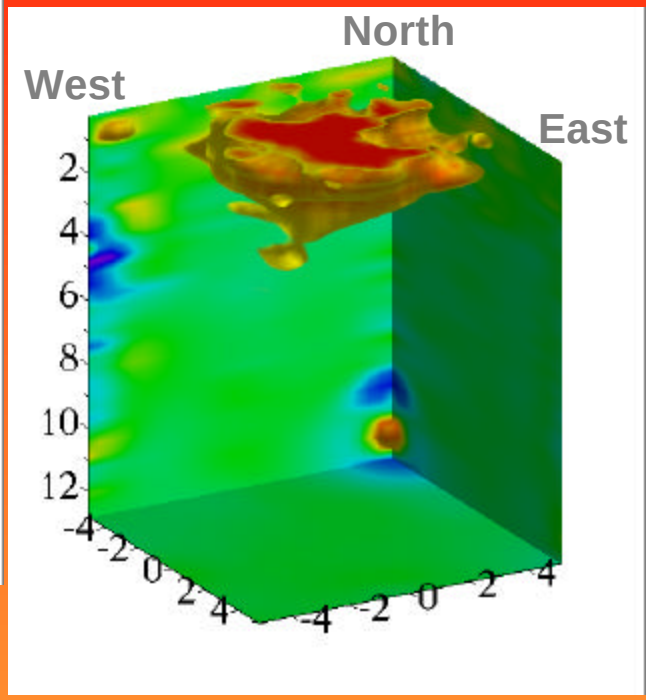
ERT Difference Results



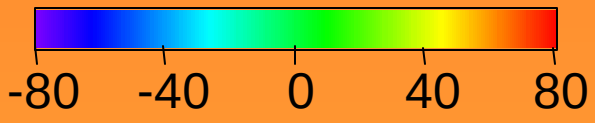
March 12 (Day 1)



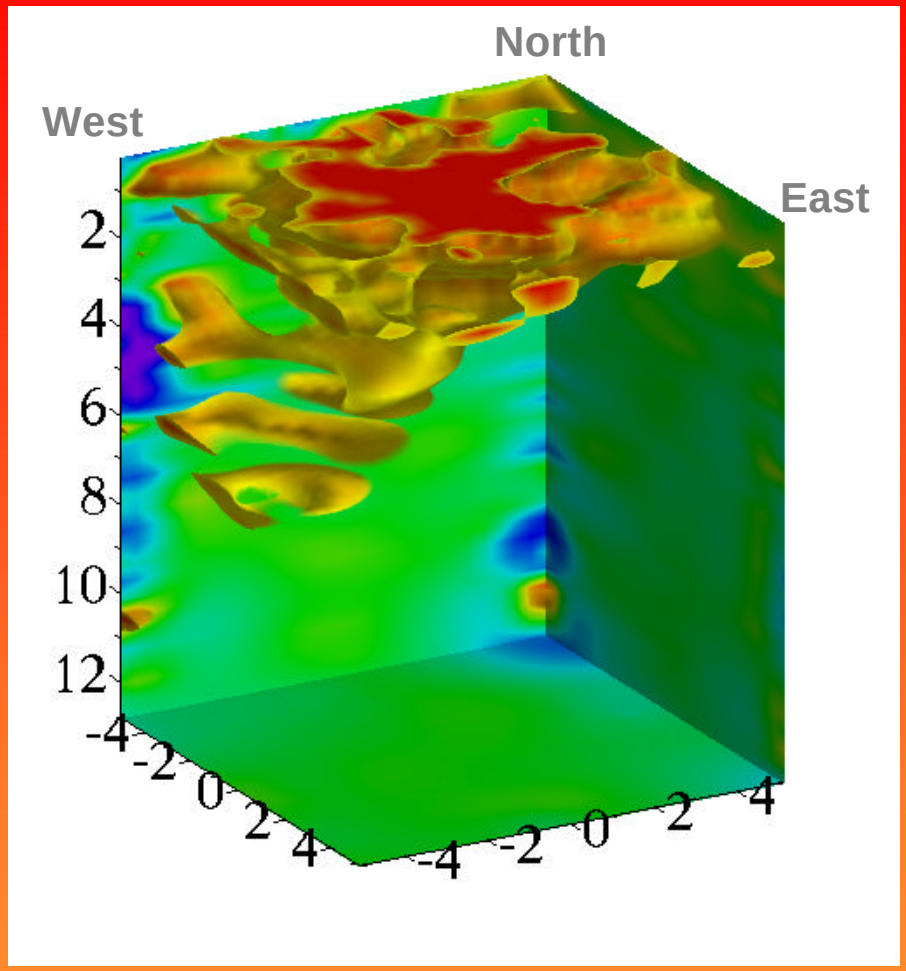
March 20 (Day 9)



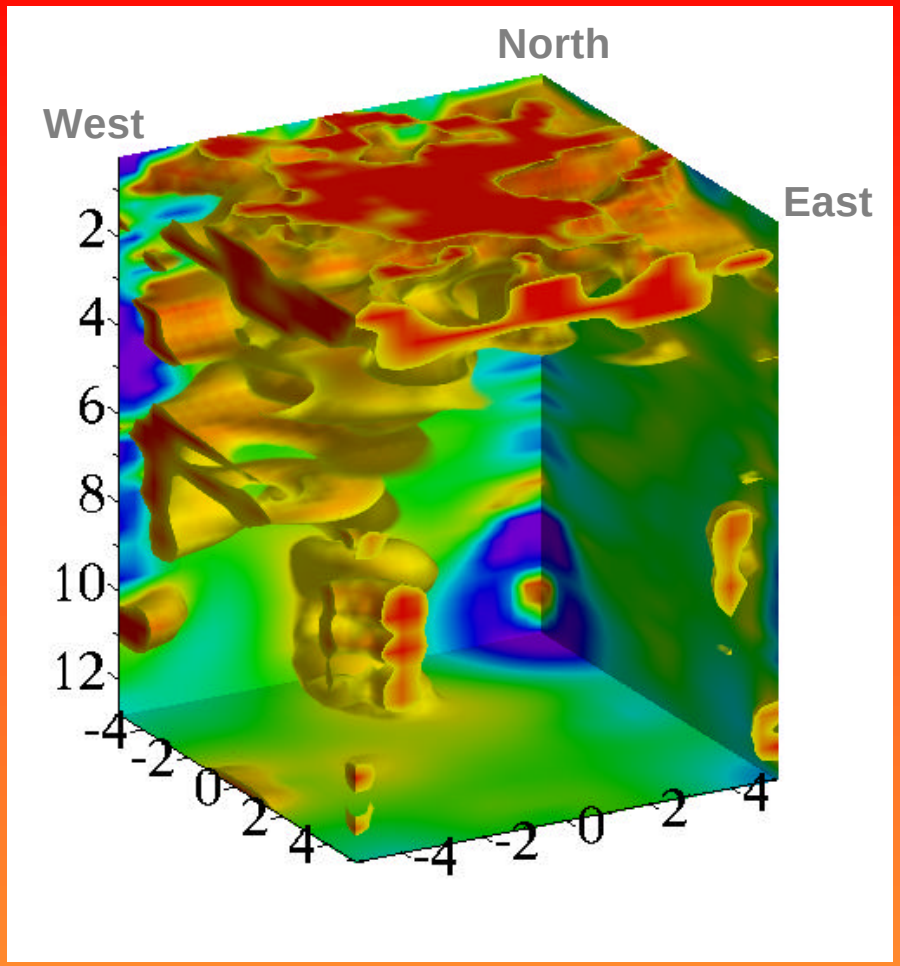
April 13 (Day 33)



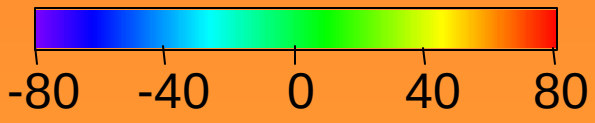
Percent Difference of Conductivity
43% cutoff for volume rendering.



May 18 (Day 68)



July 22 (Day 133)



Percent Difference of Conductivity
43% cutoff for volume rendering.